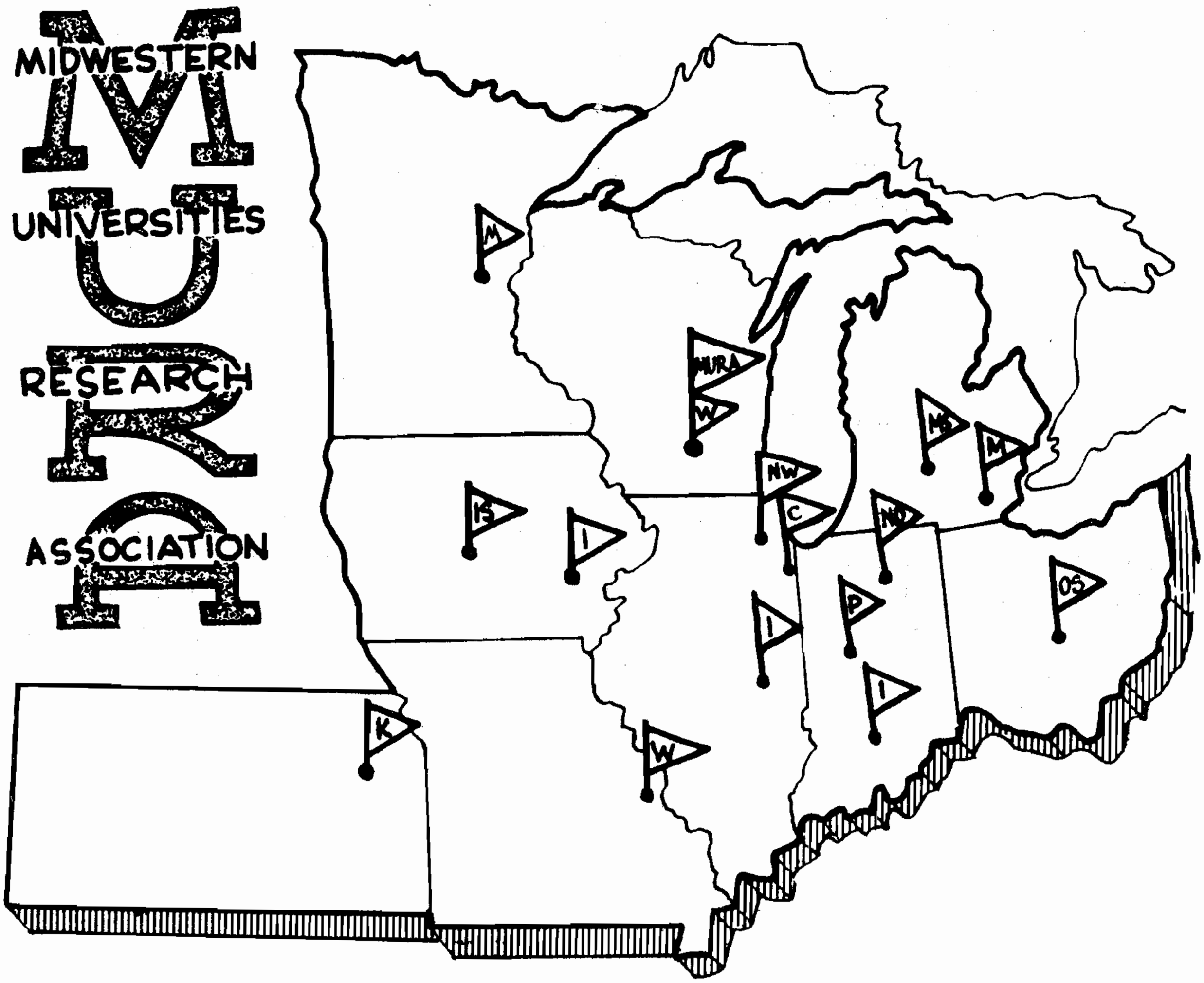


THE NUMERICAL DESIGN OF A RESONANT EXTRACTION SYSTEM

FOR THE MURA 50-MEV ELECTRON ACCELERATOR

Philip F. Meads, Jr.

REPORT **NUMBER** 712

LEGAL NOTICE

Printed in USA. Price $1.75. Available from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

MURA-712
UC-28: Particle Accelerators and
High-Voltage Machines
TID-4500 (37th Edition)

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

P. O. Box 6, Stoughton, Wisconsin

THE NUMERICAL DESIGN OF A RESONANT EXTRACTION SYSTEM FOR THE MURA 50-MEV ELECTRON ACCELERATOR

Philip F. Meads, Jr.

March 1, 1965

ABSTRACT

Detailed numerical investigations of a proposed half-integral resonant-extraction system for fixed-field, alternating-gradient synchrotrons verify the feasibility of extracting a high-quality beam from the MURA 50-MeV FFAG by this method. The half-integral resonance causes a large growth in the radial betatron amplitude, enabling the extraction of virtually the entire beam with an exceedingly small emittance. The method is complicated by the inclusion of time-varying parameters, required to maintain a relatively constant emittance and apparent source in the extracted beam over the extraction interval. Calculations were performed on an IBM-704 digital computer using measured magnetic fields.

*AEC Research and Development Report. Supported by the U. S. Atomic Energy Commission through ANL by Subcontract 31-109-38-1707.

I. INTRODUCTION

The resonant slow extraction of the beam of a fixed-field, alternating-gradient accelerator has been the object of considerable interest. The experimental advantages of a many-turn extraction system are manifold--particularly in counter experiments.. Recently, the FFAG has been proposed as an injector for large synchrotrons.[1,2] The large currents available in a FFAG of moderate energy are more than sufficient to fill existing and proposed alternating-gradient synchrotrons to their space-charge limit. The best utilization of a FFAG injector will be realized through the development of a resonant slow-extraction scheme.

The basic theory of resonant extraction using a radial half-integral resonance is presented in the paper by C. L. Hammer and L. Jackson Laslett[3] and that by Werner William Shoultz and C. L. Hammer.[4] The object of the present investigation is to develop the proposed extraction systems for the MURA 50 MeV electron FFAG accelerator by means of extensive numerical calculations using measured fields.

II. REVIEW OF LINEAR RESONANT EXTRACTION THEORY

This method of extraction consists of perturbing the magnetic field of the accelerator to force the radial betatron oscillation frequency into the nearest half-integral resonance (with the rotation frequency). When in resonance, the amplitude of betatron oscillations is no longer constrained, but rather grows with time. All orbits are expressable as the sum of two independent solutions to the radial betatron equation. In resonance, one of these solutions increases exponentially while the other one exponentially decays.

After a sufficient time interval the decaying solution may be ignored; thus all orbits, regardless of initial conditions, approach the increasing exponential solution, differing only in amplitude. In particular, every particle achieves its maximum displacement from the equilibrium orbit at the same azimuth and with nearly the same angle with respect to the equilibrium orbit. The azimuth of maximum displacement remains fixed with respect to time.

Upon reaching a certain amplitude at the chosen extraction azimuth, the particles jump a septum and emerge from the machine with virtually no spread in angle. In other words, the radial phase space of the extracted beam has an exceedingly small area. In the limit of increasing time and amplitude, the angular spread becomes arbitrarily small as the decaying exponential component of the orbits tends to zero. This property of the half-integral resonant-extraction system is particularly attractive when the extracted beam is to be injected into another accelerator.

The azimuthal envelope of the betatron motion is strongly influenced by the field perturbations. One of the goals of this study is to shape this envelope so that the maximum radial spread of the beam occurs at the desired azimuth and the beam is restrained from striking obstructions elsewhere in the accelerator.

The radial width of the beam passing the septum depends upon the growth rate at that amplitude. The larger this width, the higher is the extraction efficiency as a correspondingly smaller fraction of the beam collides with the septum. Thus, we seek to obtain a large growth rate at the septum amplitude. This too is a function of the field perturbation chosen.

The field perturbation also may shift the equilibrium orbit. If the equilibrium orbit is shifted toward the center of the accelerator, the permitted spread of the beam at that azimuth is increased; this is desirable to minimize the portion of the beam striking the septum. On the other hand, shifting the equilibrium orbit outward at the extraction amplitude brings the entire beam closer to the exterior of the machine, thus reducing the bending field required to extract the beam.

Another consideration involved in choosing the shape of the perturbing field is that of bringing the beam into the unstable region. Two approaches are considered: (1) accelerating the beam into a dc perturbing field, and (2) stacking the accelerated beam and then pulsing the perturbing field to bring the beam out. The relative merits of these two approaches are discussed subsequently.

All of the preceding arguments are based on a linear theory where in fact FFAG accelerators are decidedly nonlinear. The nonlinear effects are particularly important in resonant extraction studies because large radial beam amplitudes are generated. Thus we surely must carefully consider the effect of the nonlinear terms in selecting the shape of the perturbing field imposed to extract the beam. It is for this reason that numerical studies are particularly valuable.

Additional considerations involved in the selection of the type of field perturbation are those of practicality. We would prefer that the perturbing coils be inexpensive and easy to construct. The field must not be so complicated that overly critical adjustments are required for successful operation.

It is imperative that the perturbation chosen have no adverse effect upon the acceleration of the beam to the extraction energy lest there be no beam to extract.

III. THE NUMERICAL METHODS APPLIED TO THE PROBLEM

A. Equations of Motion

Individual trajectories are calculated by integrating the following differential equations with θ as the independent variable:

$$\frac{dr}{d\theta} = r\, p_r\, (p^2 - p_r^2)^{-\frac{1}{2}}\,, \qquad \frac{d\,p_r}{d\theta} = (p^2 - p_r^2)^{\frac{1}{2}} - r\, B_z\,,$$

$$\frac{dz}{d\theta} = r\, p_z\, (p^2 - p_r^2)^{-\frac{1}{2}}\,, \quad \text{and} \quad \frac{d\,p_z}{d\theta} = z\left[r\,\frac{\partial B_z}{\partial r} - p_r\,(p^2 - p_r^2)^{-\frac{1}{2}}\,\frac{\partial B_z}{\partial \theta}\right]. \quad (1)$$

These equations depend upon the magnetic field in the median plane of the accelerator and its derivatives with respect to r and θ. The equations for radial motion are exact in the median plane; those for the vertical motion are truncated to linear terms. Inasmuch as the extraction mechanism depends only upon the radial behavior, the above equations suffice to determine the feasibility, apart from possible vertical beam blow-up, of particular schemes of field perturbation. Having chosen one such scheme, the stability of the vertical motion may be investigated by using higher order approximations to the vertical equations of motion. This choice of independent variable and truncated equations significantly reduces computation time while retaining the needed accuracy.

B. Orbit Code

The code developed for these calculations is an extensive modification of Oak Ridge Orbit Code No. 1482[5], Oak Ridge General Orbit Code (GOC)[6], and the Bevatron Orbit Code.[7] The unperturbed median-plane magnetic field and its azimuthal derivative are stored for discrete values of r and θ. The perturbing fields are added to the unperturbed field. It is assumed that each magnet in the MURA 50 MeV accelerator may be perturbed independently of any other magnet. Measurements confirm this simplification; perturbing one magnet has a very minor effect on neighboring magnets providing coil currents are trivially readjusted for those magnets. Given a particular type of perturbation, we measure the azimuthal and radial profiles of the perturbing field. The azimuthal profile (and the azimuthal profile of the θ derivative) from the magnet center to center of the intermagnet drift space is stored; we assume this profile to be the same, regardless of which magnet is "bumped." The radial profile is calculated from a theoretically derived analytic expression that contains parameters chosen to agree with actual measurements. The analytic expression allows us to vary a number of parameters pertaining to the perturbations. As θ is the independent variable in the equations of motion, this method of prescribing the field bumps retains flexibility while taking advantage of the fact that the fields are required only at discrete values of θ.

C. Magnetic Field Format

The measured median plane fields are available on magnetic tape in the form of dimensionless Fourier harmonics at discrete radii. These (JAN) tapes are produced by Fourier analysis of field data measured on a grid of 2.0 cm (r) by 2.25° (θ). In order to produce the field and azimuthal derivative tapes required by the orbit code (same format as the Oak Ridge and Bevatron codes), an IBM-704 FORTRAN code, called "GETB", was written. This code synthesizes the fields and azimuthal derivatives from the JAN tape harmonics, adjusting the azimuthal and radial intervals as specified; four-point interpolation formulae are used. GETB may adjust the radial exponential, $(r/r_o)^k$, in the fields (presently $k = 9.25$) as is possible in the real accelerator. Field perturbations may be added while producing the field tapes though this has not been done. Supplemental output from GETB consists of optional listings of harmonics and synthesized fields and derivatives, radial field profile plots, and azimuthal profile plots (using a CALCOMP plotter attached to an IBM-1401 computer). For the present work, a radial interval of 0.5 inch and an azimuthal interval of $\left(\frac{360}{512}\right)^{o}$ were specified; the fields are stored between $r = 72.0$ in. and $r = 82.0$ in, and θ between 0^{o} and 180^{o}. The first 48 azimuthal Fourier harmonics of the fields are included. This method of obtaining the median plane field data was selected because the resulting fields are "smoothed" by truncating the (superfluous) harmonics beyond the 48th. "Smoothed" fields are highly desirable where interpolation is required.

D. Orbit Code Capabilities

The orbit code, MURA code no. 323, uses the Runge Kutta method, as contained in the MURA MURKY7 Runge Kutta integration routine,[8] to solve the equations of motion. Upon demand the code integrates the linear radial and vertical motion about a selected orbit. It may also be directed to calculate the orbit arc length and the Courant-Snyder functions $\alpha(s)$, $\beta(s)$, $\psi(s)$.[9] Equilibrium orbits, radial and vertical tunes, and other linear properties of equilibrium orbits may be calculated with or without field perturbations. Linear dispersive properties about a given orbit are also included.

Primary output consists of a listing of r and p_r at selected azimuths for the orbit being calculated. Radial and vertical transfer matrices (with dispersive terms included in the radial matrix) and radial and vertical betatron amplitudes (linear) are also listed if desired. An azimuthal plot of the betatron envelope may be specified.

At 256 Runge Kutta steps per revolution, the code requires but 15 seconds to integrate a single orbit (radial equations only) once around the accelerator. Approximately one minute is required to integrate all sixteen equations of motion (linear radial and vertical betatron motion, linear dispersion, and other functions mentioned above) through one revolution. The code operates at intermediate speeds when called upon to produce intermediate amounts of information.

The orbit code calculates the perturbing field during the integration process, using stored data defining the magnets to be perturbed and the strength and shape of the perturbing field at each magnet. The time required to calculate the field perturbations is small compared to the time required to perform the integration.

The code is equipped to Fourier analyze the total field obtained by adding the field perturbations to the stored unperturbed field. These azimuthal harmonics may be listed at selected radii. Inasmuch as the code cannot calculate the non-linear effects due to quadratic and higher terms in z and p_z that occur in the exact equations for both radial and vertical motions, it may be directed to prepare a JAN (field harmonic) tape that contains the generated perturbations. This JAN tape may then be used in other, slower codes that integrate the equations of motion through higher terms.

The accuracy of the code has been verified by first plotting the developed fields to observe their general character and then by calculating the radial and vertical tunes as a function of radius. These tunes agree with both experimental observations and calculations performed on other codes.

IV. THE COMPUTATIONAL PROCEDURE

We followed a definite sequence in evaluating the desirability of each particular field perturbation scheme. First radial and azimuthal profiles of the perturbing field were evaluated for several coil geometries. The field induced by a single conductor placed across a magnet at a constant radius has been derived by J. Van Bladel for the case where the conductor lies on the pole face[10] and by S. C. Snowdon for a more general case.[11] Measurements on actual magnets for the former case were performed under the direction of M. Shea. Electrolytic tank measurements were conducted by E. Rowe on other possible conductor designs. Field perturbations used in the code were chosen to coincide with experimental measurements.

A. Harmonics

Given a particular perturbation model, the code was directed to azimuthally Fourier analyze the perturbed field. We then looked for the relative enhancement of both desirable and undesirable harmonics.

B. Tunes

Having selected a likely field perturbation, we next determined equilibrium orbits for a range of momenta to obtain the betatron frequencies as functions of momentum. Of particular interest was the effect of a given perturbation upon the $\nu_x = 4.5$ stopband and associated tune shift. From these runs we determined that momentum where small radial oscillations are just unstable.

C. Integrated Trajectories

The actual orbits in the presence of the perturbation were next calculated. Starting well inside the bump, we assumed that the beam is uniformly distributed in betatron-oscillation phase with a radial betatron amplitude of 0.1 inch (somewhat larger than measured). We accelerated rapidly to a momentum just below that where the beam goes unstable. This rapid acceleration is justified because the beam shape is nearly independent of momentum until the beam becomes unstable. Digital calculations with two rates of acceleration showed no significant differences. We accelerated the beam into the unstable region over approximately one hundred turns; the acceleration, about 300 times the normal acceleration rate in the actual accelerator, was spread uniformly around the machine. As the beam approached the unstable momentum, the acceleration was slowed down to about thrice that actually experienced by the beam. In most cases the growth rate increases rapidly with the number of turns in the unstable region; thus a more realistic rate of acceleration was called for here.

We chose to calculate nine orbits simulataneously. The first was the reference orbit, initially an equilibrium orbit. About this reference orbit we calculated the linear transfer matrices for both planes and the dispersive terms. The other eight orbits all had an initial betatron amplitude of 0.1 in (the maximum initial amplitude) and were uniformly distributed in initial phase. By comparing the beam envelope determined by the eight bounding orbits with the envelope determined by the radial transfer matrix, we had an immediate measure of the effect on the beam of various nonlinearities. The trajectories and phase space curves plotted from these runs provided the bulk of information for determining the effectiveness of each bump configuration.

For two configurations of particular interest, the above set of orbits were supplemented with others having smaller initial amplitudes. These orbits require more time to reach extraction amplitude; we desired to assure ourselves that they would be extracted at the same azimuth and direction as the larger initial amplitude orbits.

Vertical motion was examined for a number of orbits having large radial excursions. These orbits allowed us to determine whether bad vertical growth occurred in the process of extraction. Due to the linear nature of the vertical equations, resonant effects caused by quadratic and higher powers in z and p_z would not show up in these runs. Before committing ourselves to a particular extraction scheme, we would thoroughly check the vertical stability using the complete equations; this we have not yet done.

We next discuss several perturbation schemes examined and the results obtained.

V. DISCUSSION OF CALCULATIONS AND RESULTS

Two schemes of half-integral resonant extraction were considered. Most of the effort was applied toward designing a continuous extraction system that would extract the beam at a rate controlled by the rate at which the beam is accelerated into the unstable region of the machine. A second approach, of particular interest where injection into a larger accelerator is the object of the extraction, is to extract the beam over a number of turns by means of a pulsed bump.

A. Merits of Pulsed Extraction

The pulsed-bump scheme is attractive where we desire to extract the entire beam during the injection cycle of the larger machine. Since the perturbing coils are pulsed, much greater freedom exists in their design, for we are not concerned about their effect upon the field inside the stacked-beam radius.

A beam for injection ought to be reasonably constant in radial and angular spreads and in current during the extraction period. This is impossible with the pulsed-bump half-integral resonant extraction method. In order to maintain a constant beam width, we require that the growth rate be constant which, in turn, requires a constant bump magnitude. The radial width of the beam is given by

$$W = x_s \left[e^{4\pi\mu} - 1 \right] \tag{2}$$

where x_s is the distance from the reference orbit to the septum, and μ is the growth rate; note that the radial width is maximized at the azimuth of maximum betatron amplitude. Here we assume that all betatron trajectories satisfy the following equation (Ref. 4) which neglects the decaying exponential component:

$$x(\theta) = A\,\Phi_1(\theta)\,e^{\mu\theta}. \tag{3}$$

Since the beam is in a half-integral resonance, a particle achieves a maximum displacement at intervals of two revolutions. The envelope function, $\Phi_1(\theta)$, is periodic in θ with a period of 4π. With a constant growth rate and an assumption of uniform initial distribution of the beam within the bounding radial phase-space curve, it is easy to show that the number of particles extracted on the n^{th} turn is $N_o e^{-2\pi\mu(n-1)}$ where N_o is the number extracted on the first turn.

We can approach a constant beam current by programming the bump to increase with time, thus increasing the growth rate; this has the disadvantage of increasing the beam width from turn to turn. However, if a uniform current is achieved by programming the bump magnitude, then the phase-space area of the extracted beam will also be constant. It is conceivable that a series of lenses could be programmed to alter the magnification of the extracted beam in such a way that the beam appears to occupy the same region of phase space during the extraction interval. The same external magnet system would have to be programmed to compensate for any shift in extraction angle caused by the increase in bump magnitude during the extraction period.

B. Continuous Extraction

The continuous extraction method avoids these problems for two reasons. The beam size is necessarily constant with time because the bump and septum location are not varied. We are able to control the current of the extracted beam by controlling the acceleration of the stacked beam with time. (It is not possible, by the way, to accelerate synchronously the stacked beam in the 50 MeV electron FFAG because of the very large energy spread, but the purpose of this study is to demonstrate, numerically, extraction on that machine.) A disadvantage of continuous extraction is that of small growth rates. At the momentum where the beam is first unstable, the growth rate is zero. During succeeding turns the beam is gradually accelerated further into the unstable region; thus the growth rate increases with each turn. It is our experience that the beam never achieves a large growth rate because it reaches the septum amplitude before it has ventured very deeply into the unstable region. The growth rate is enhanced somewhat by the nonlinear tune shift that causes the beam to move further into the unstable region as its amplitude increases. The efficiency of the extraction system depends upon the ratio of septum width to beam width (Eq. (2)).

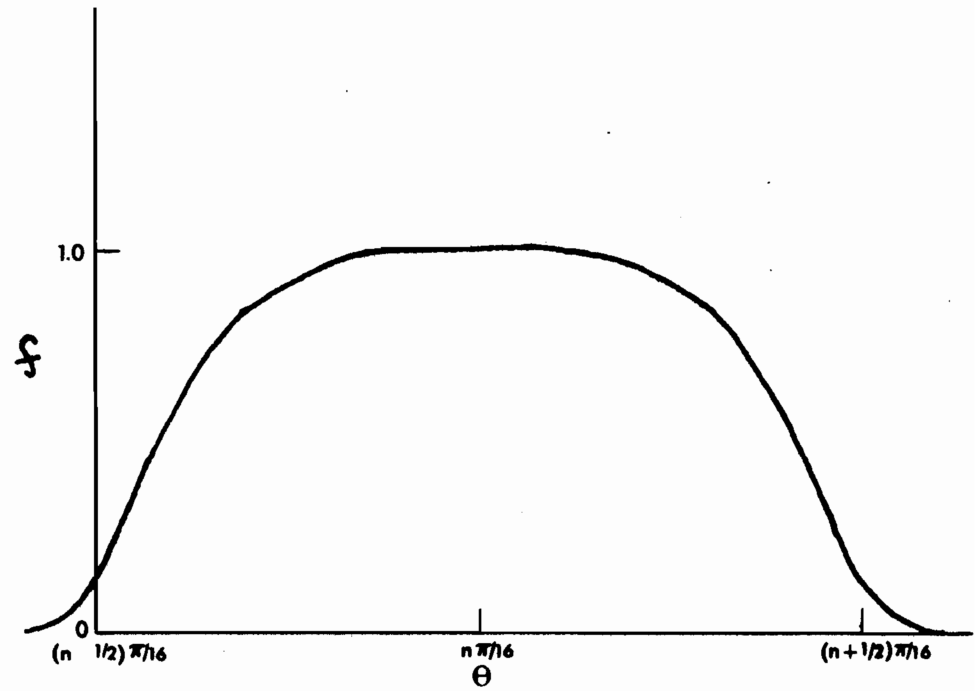
1. Bump Shape for Continuous Extraction

The analytic work by Van Bladel[10] and Snowdon[11] and the measurements by Shea and Rowe mentioned above resulted in the following expression for the perturbing field:

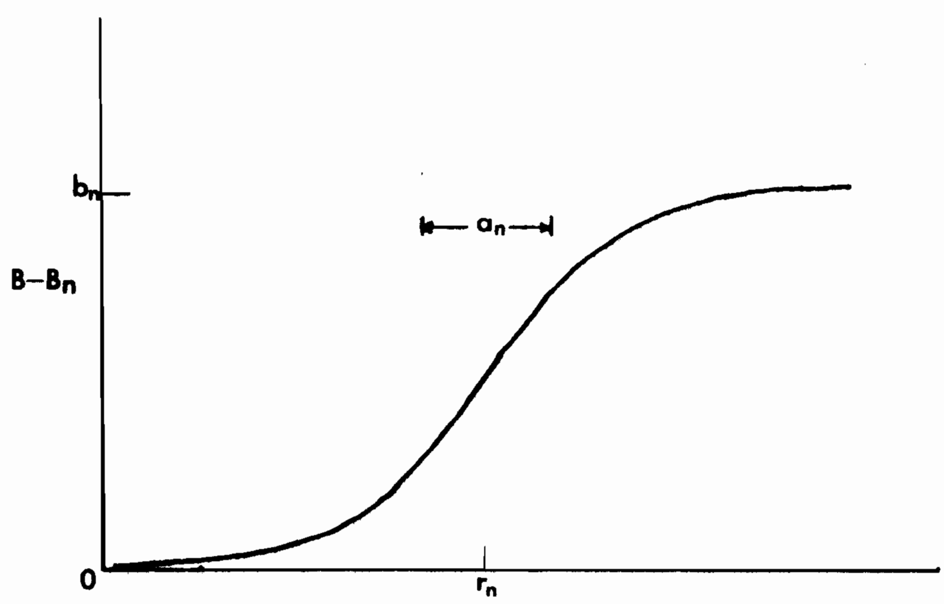
$$B\,(r,\ \theta) = B_o\,(r,\ \theta) + \sum_{n=1}^{32} \frac{1}{2}\, b_n\, f\,(\theta - n\,\frac{\pi}{16}) \left\{1 + \tanh\,(\frac{r - r_n}{a_n})\right\} \quad . \qquad (4)$$

Here r_n is the radius of the perturbing coil, b_n is the magnitude of the bump field at a large distance beyond the coil, $f\,(\theta - n\,\frac{\pi}{16})$ is the azimuthal profile of the bump field as shown in Fig. 1, and a_n is the characteristic radial width over which the bump rises. With the conductor lying on the magnet pole face at the extraction radius, a numerical agreement with the measured field is obtained for $a_n = 1.93$ inches; this figure may be reduced by lifting the coil from the pole face. We show in Fig. 2 a typical radial profile of the bump field. In all calculations, we assume that correction windings are utilized to eliminate effects of the bump inside the extraction region and in unperturbed magnets. Perturbing fields are assumed azimuthally symmetric with respect to the center of the "bumped magnet."

A power series expansion of the bump magnitude as a function of r yields a gradient at the bump center of magnitude b_n/a_n and nonlinear terms



Measured Azimuthal Profile of Bump Field
Fig. 1



Typical Radial Profile of Bump Field
Fig. 2

amounting to approximately 10% at $r = r_n \pm \frac{2}{3} a_n$. The theory of half-integral resonance is based on the gradient; nonlinear terms lie outside of this theory and are generally not beneficial. We can achieve a much larger gradient by lifting the bump coil away from the pole face (thereby decreasing a_n), but this reduces the maximum betatron amplitude attainable prior to encountering large nonlinear effects.

2. Choice of Magnets to Perturb

We choose the magnets to be perturbed primarily to enhance certain azimuthal harmonics in the field while depressing others. The coils and septum required for the extraction system are restricted to those magnets where sufficient room exists. Furthermore, we seek to minimize the number of magnets to be perturbed. In this study, we perturb either one magnet or three magnets. The theory developed in references 3 and 4 provides the criteria for choosing those harmonics to be enhanced and those to be depressed. Other harmonics must be avoided as they feed nonlinear resonances near the operating point (for example, the $3\,\nu_x = 13$ and $3\,\nu_z = 8$ resonances).

The $9\,\theta$ harmonic is needed to open up the $\nu_x = 4.5$ stopband and the zeroth harmonic to shift the tune into this stopband. The choice of bumps is restricted to those retaining symmetry for reflection about $\theta = 0$ in order to eliminate the introduction of $\sin n\,\theta$ terms which are shown in the references to be generally undesirable. The harmonics having the greatest effect in peaking the ampltiude at the extraction azimuth are those closest to the resonant frequency, namely the $4\,\theta$ and $5\,\theta$ harmonics. Further discussion of the relative merits of other harmonics is contained in the papers by Hammer *et al*.

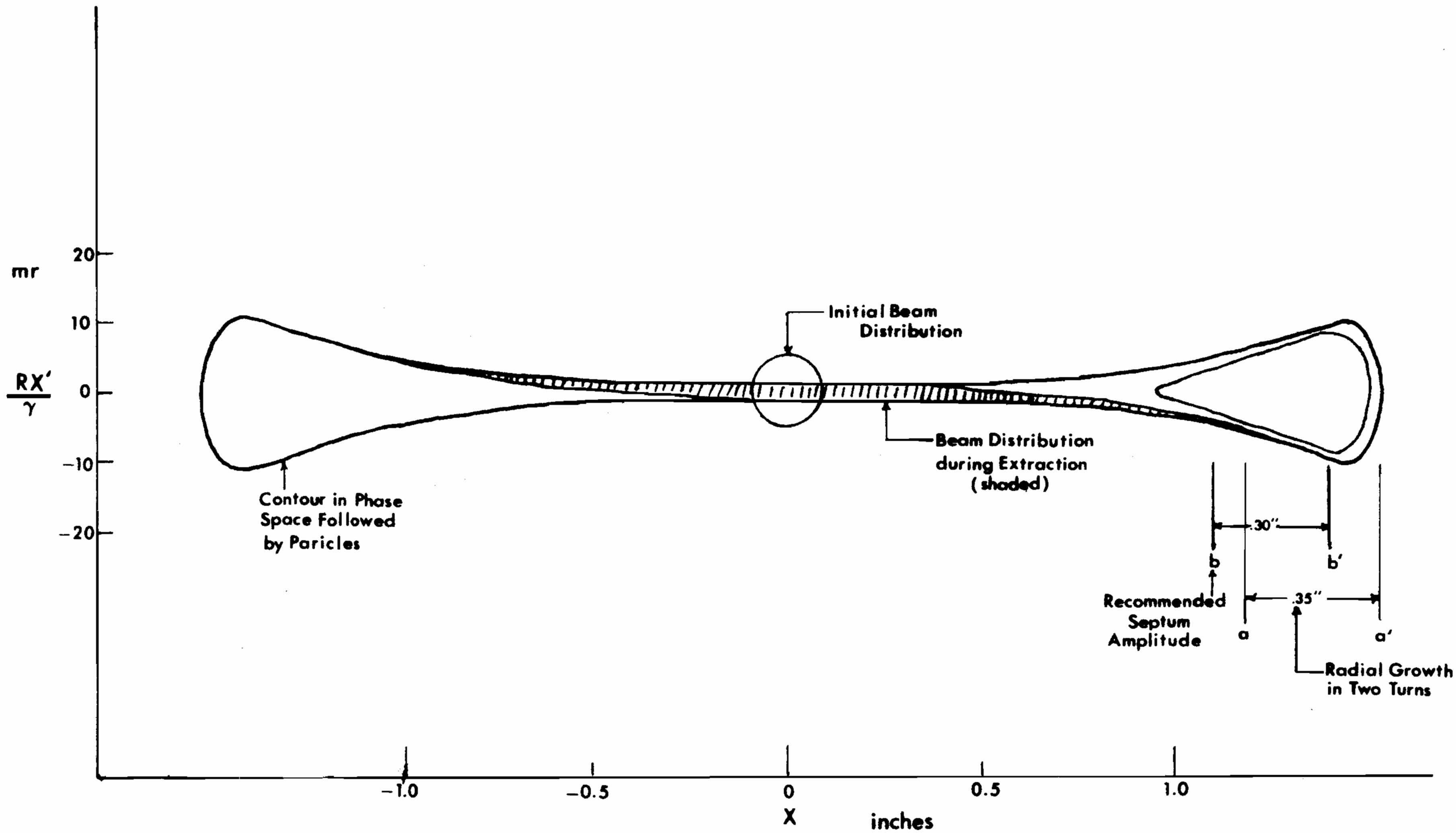
We emphasize here that there is an advantage in not having the equilibrium orbit peak at the extraction azimuth, namely that of accommodating larger betatron amplitudes; our experience indicates that this is of little interest as nonlinearities already limit the useful maximum betatron amplitude. We require a large growth rate, peaking of the betatron amplitude at the extraction, and containment of the beam elsewhere. In practice, we do not specify the extraction azimuth prior to the calculations.

a. Three Magnet Bump--Configuration "A". Our first series of calculations were based on a perturbation consisting of three symmetrically placed magnets, each identically bumped (except for sign) with a single conductor of low current placed on the magnet pole face in the azimuthal direction. We chose the magnets to enhance the $4\,\theta$, $9\,\theta$, and $10\,\theta$ harmonics (Ref. 4, p. 15). The currents in the bump conductors were on the order of 20 amp. Such low currents induce small harmonics, nevertheless sufficient for extraction. One particular case had perturbations of 0.02% of the main field applied at one positive magnet (field increased) and two negative magnets at $\theta = \pm\frac{5\pi}{16}$ (negative bumps); this bump induces harmonics $4\,\theta$, $9\,\theta$, and $10\,\theta$ of magnitudes -0.06%, -0.07%, and -0.04%, respectively, referred to the dominant $16\,\theta$ harmonic. A growth of 0.10" occurs at a betatron amplitude of 0.4"; the extraction energy is 44 MeV for this example. Growth beyond 0.4" is precluded by nonlinear effects that move the beam off resonance at this amplitude. The betatron envelope had peaks at several locations around the machine nearly equal to that at the extraction azimuth of $\theta = 180^{\circ}$; this is not surprising because many other harmonics are induced by the chosen bump. Such a small bump is inadvisable because the induced harmonics are less than an order of magnitude greater than the corresponding harmonics in the unperturbed field.

b. Single Magnet Bump. From a cost and simplicity standpoint, a most attractive perturbation scheme is one where a single magnet is bumped with a single conductor. This we did by depressing the field on a positive magnet by 70 gauss and accepting the resulting harmonics. This bump induces all harmonics with low harmonics having magnitudes of the order of 0.1% of the 16 θ harmonic. A growth of 0.10" is achieved at an amplitude of 0.5". Non-linearities move the beam away from the resonance at larger amplitudes.

c. Three-Magnet Bump--Configuration "B". Dr. Hammer has proposed a three-magnet bump designed to accentuate the 4 θ and 9 θ harmonics without increasing the 3 θ, 5 θ, 11 θ, and 13 θ harmonics. The field of the magnet at $\theta = 0$ is increased by 112 gauss while those at $\theta = \pm \frac{3\pi}{4}$ are perturbed with an opposing field of 79 gauss; the ratio of the two bump fields is equal to the cosine of 135°. With this bump, we obtain a 4 θ component of -0.24% and a 9 θ component of -0.20%. A maximum growth of 0.35" is obtained at a (septum) betatron amplitude of 1.3". This amplitude occurs at six locations around the machine, but the equilibrium orbit perturbation places the maximum radial excursion at $\theta = 180^{\circ}$. Extraction with this bump occurs at 38.6 MeV.

Figure 3 is a radial phase-space plot showing closed phase-space orbits and the growth of the beam envelope. The beam envelope is well approximated by an ellipse until it grows beyond 0.8" in amplitude. The growth rates from the linear equations agree with those calculated from the exact equations



Beam Distribution With Continuous Extraction

Fig. 3

up to this amplitude. Beyond this amplitude, the growth is more rapid than indicated by the linear equations. However, at an amplitude of 1.4", the beam turns over in phase space and grows very little beyond this amplitude. We obtain the maximum growth of 0.35" by placing the septum at "a" (Fig. 3); the extracted portion of the beam will then be that between "a" and "a'", clearly very distorted due to nonlinearities. By moving the septum to a slightly smaller amplitude as indicated by "b", we obtain a high quality extracted beam 0.30" wide. This latter choice of septum location yields a beam that will be much more manageable at little expense in extraction efficiency. The radial emittance is of the order of 1% of the radial phase area of the beam prior to resonant growth. Calculations show that particles with smaller initial amplitudes grow to occupy the same region of phase space at the extraction amplitude. We would expect to extract 97% of the beam with a 0.01" thick septum, assuming perfect septum alignment.

Linear vertical motion was calculated about that radial trajectory having the largest betatron amplitude. The amplitude of vertical oscillations was observed to approximately double by the time the particle was extracted. The investigation would not show coupling effects involving powers of z and z' higher than quadratic in the Hamiltonian.

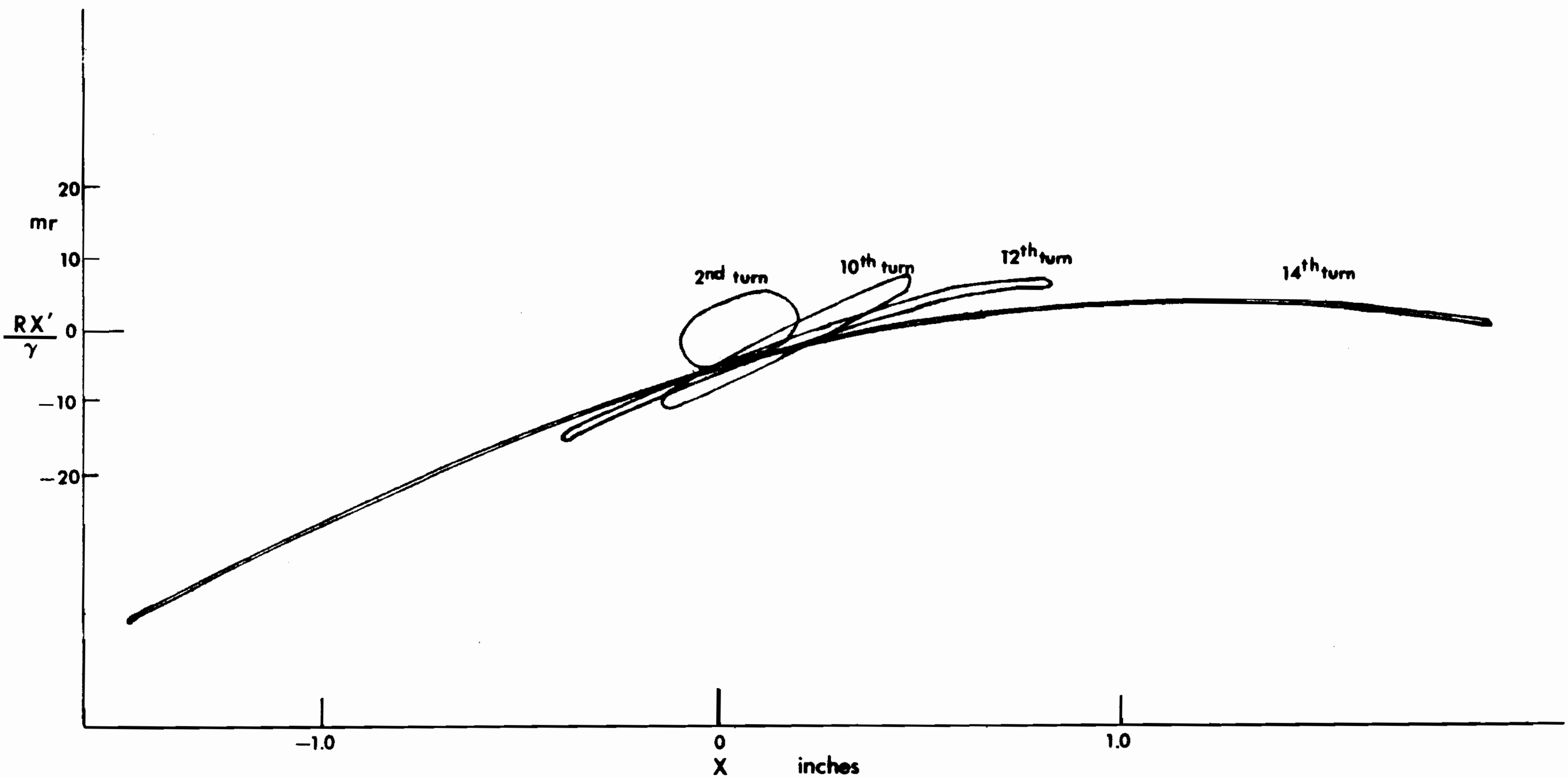
d. High-Gradient Three-Magnet Bump. We investigated a bump identical in every respect to that described above except for a gradient ten times as large (a_n = 0.2"); the magnitudes of the bump fields were not changed. As mentioned in Section B. 1 above, we expect a much larger growth rate and much larger nonlinear effects with this bump. A maximum growth of 0.89" over two turns is obtained. If we insist upon a high quality extracted beam (little distortion), then we are limited to a growth of 0.6". The nonlinearities are not harmful up to this point; however, there is virtually no agreement between the growths calculated from the linear equations and those observed using the exact equations. Even at small amplitudes, the beam envelope in phase space is badly distorted from the initial elliptical shape.

C. Pulsed Extraction Calculations

A number of bump shapes were investigated for a pulsed extraction system. We assume the initial existence of a stacked beam in an unperturbed field. The bump is then turned on slowly over approximately 20 turns of the beam. In each case, the beam was observed to be extracted very rapidly, well before the bump has been brought to full amplitude.

The steep-gradient three-magnet bump described in the last section proved to be quite undesirable for the pulsed extraction system. Large beam growth occurred in a few turns, but no phase lock-in was observed. This lack of phase lock-in demonstrates that the betatron motion is not dominated by the increasing exponential component; the nonlinear terms clearly determine the character of the oscillations.

Using a pulsed extraction system, we are released from the limitation that the perturbation applied not affect the motion inside the extraction region. Most of the observed nonlinearities can be eliminated by using a pure gradient bump to replace that with the radial dependence illustrated in Fig. 2. We investigated a number of configurations consisting of gradient bumps only. One of these was a single gradient bump for which $\frac{1}{B}\frac{\partial B}{\partial r} = 1.0 \text{ inch}^{-1}$; this bump was turned on smoothly over a duration of 20 turns of the circulating beam. The phase-space profile of the beam is plotted in Fig. 4 at the 2nd,



Resonant Beam Growth With Single Magnet Pulsed Bump
Shown 22 1/2° Downstream
From Bump

Fig. 4

10th, 12th, and 14th turns from the start of the bump turn-on. A betatron amplitude of 0.7" is achieved in 12 turns with the beam essentially elliptical in phase space. The amplitude increases by 1.0" in the next two turns with the beam now lying on a gentle curve as shown in Fig. 4. Thus there appears to be no difficulty in extracting virtually the entire beam. The fact that the bump continues to increase in magnitude while beam is being extracted causes the beam to move in phase space during the extraction process; we assume that the external optical system compensates for this. The energy of the extracted beam just described is 35.7 MeV; similar results are obtained at higher energies except that the beam suffers greater distortion owing to the poorer guide field at these energies.

As explained earlier, a rising bump field is required if we are to maintain an approximately constant extraction rate.

Several schemes were examined in which the bump was turned on within a fraction of a turn (as is done for the existing single-turn extraction method). The rapid change in magnetic field causes the entire beam to oscillate wildly and leave the machine before an appreciable growth in width has occurred. This could be improved by accurately centering the bump so a particle on the equilibrium orbit would experience no change in the magnetic field. However, this method would result in a beam differing slightly from a single-turn extracted beam owing to the aforementioned exponential decay in beam current.

D. Conclusions

Digital computer studies of several proposed half-integral resonant-extraction schemes reveal that it is possible to extract virtually all of the beam with only a very small portion striking the septum. Both the continuous extraction and pulsed extraction systems can be designed to yield an external beam of high optical quality with a very small emittance. In both systems, a septum location that results in a beam of low distortion is to be preferred to the location yielding the minimum beam loss on the septum. The perturbations required to extract the beam are provided by coils of simple construction. Control of extraction rate is provided by a programmed rate of acceleration in the continuous system and by a programmed bump rise in the pulsed system; a reasonably uniform extraction rate over a large number of turns is desirable in either case. With the pulsed system, an external array of lenses must be programmed to compensate for the change in the apparent source of the extracted beam during the extraction interval. The perturbations should approximate a pure gradient bump as nearly as possible, consistent with other restrictions, including the requirement for an adequate growth rate. We believe that the half-integral resonant extraction system is preferable to those depending upon higher order resonances on grounds of simplicity in construction and the minimal perturbation field magnitudes required.

REFERENCES

1. S. C. Snowdon, "Study of a 500 MeV High Intensity Injector", MURA Report No. 700, September 8, 1964 (unpublished).

   E. M. Rowe, Addendum to MURA-700.

2. E. M. Rowe, "On the Suitability of the 500 MeV FFAG Synchrotron as an Injector for the AGS and ZGS", MURA Report No. 701, September 25, 1964 (unpublished). Also E. M. Rowe, Addendum to MURA-700.

3. C. L. Hammer and L. Jackson Laslett, "Resonant Beam Extraction from an AG Synchrotron," Rev. Sci. Instr. 32, 144 (1961).

4. Werner William Shoultz and C. L. Hammer, "Resonant Beam Extraction in Fixed Field Alternating Gradient Accelerators," MURA Report No. 666, January 2, 1963 (unpublished).

5. M. M. Gordon, T. A. Welton, T. I. Arnette, and H. C. Owens, "An IBM-704 Code for Determining Equilibrium Orbits and Properties of Small-Amplitude Oscillations in Cyclotron Fields", Oak Ridge National Laboratory Laboratory Report ORNL 59-11-2 (Oak Ridge, Tenn.) November 17, 1959 (unpublished).

6. H. C. Owens and T. A. Welton, "An IBM-704 Code for Studying Particle Orbits in Cyclotron Fields", Oak Ridge National Laboratory Report ORNL 59-11-3 (Oak Ridge, Tenn.) November 2, 1959 (unpublished).

7. P. F. Meads, Jr., "II. Ion Optical Design of High Quality Extracted Synchrotron Beams with Application to the Bevatron", Lawrence Radiation Laboratory (Berkeley) Report UCRL-10807, May 1963 (unpublished).

8. P. F. Meads, Jr., "Floating Point Runge-Kutta MURKY7", MURA Report No. 704, September 10, 1964 (unpublished).

9. E. D. Courant and H. S. Snyder, "Theory of the Alternating-Gradient Synchrotron," Annals of Physics 3, 1 (1958).

10. J. Van Bladel, "Magnetic Fields for Extractor Magnet", MURA TN-438, October 18, 1963 (unpublished).

11. S. C. Snowdon, "DC Extractor Magnet Field Perturbation Bump", MURA TN-516, November 10, 1964 (unpublished).