

## Impure Mathematics

Aunty Derivative

*Wherein it is related how that Polygon of Womanly Virtue, Polly Nomial (our heroine) is accosted by that Notorious Villain Curly Pi, and factored (oh, horror).*

Once upon a time (1/T) pretty little Polly Nomial was strolling across a field of vectors when she came to the boundary of a singularly large matrix.

Now Polly was convergent, and her mother had made it an absolute condition that she must never enter such an array without her brackets ([ ]) on. Polly, however, had changed her variables ( $x \rightarrow \tilde{x}$ ) that morning and was feeling particularly badly behaved. She ignored this condition on the basis that it was insufficient, and made her way amongst the complex elements.

Rows and columns closed in on her from all sides. Tangents approached her surface. She became tensor ( $g^{ij}$ ) and tensor ( $g_{ij}$ ). Quite suddenly, two branches of a hyperbola ( $xy = a$ ) touched her at a single point. She oscillated violently ( $\sin \frac{1}{x}$ ), lost all sense of directrix, and went completely divergent. As she reached a turning point, she tripped over a square root ( $\sqrt{\quad}$ ) protruding from the erf ( $\int \exp(-x^2) dx$ ) and plunged headlong down a steep gradient ( $\nabla$ ). When she rounded off once more, she found herself inverted<sup>-1</sup>, apparently isolated, in a non-euclidean space ( $\Gamma_{jk}^i \neq 0$ ).

She was being watched, however. That smooth operator, Curly Pi ( $\pi$ ), was lurking inner product ( $\cdot$ ). As his eyes devoured her curvilinear coördinates ( $h_i$ ), a singular expression crossed ( $\times$ ) his face. He wondered, was she still convergent? He decided to integrate improperly ( $\int_0^\infty$ ) at once.

Hearing a common fraction behind her, Polly rotated and saw Curly Pi approaching her with his power series extrapolated. She could see by his dissipative terms ( $\dot{x}$ ) that he was bent on no good.

“Arcsinh! ( $\sinh^{-1}$ )” she gasped.

“Ho, Ho,” he said. “What a symmetric little asymptote you have. I can see your angles ( $\angle$ ) have lots of secs (”).”

“Oh, sir,” she protested, “keep away from me. I haven’t got my brackets on.”

“Calm yourself, my dear,” said our suave operator. “Your fears are purely imaginary.”

“ $i, i$ ,” she thought, “perhaps he’s normal, but homologous.”

“What order are you?” the brute demanded.

“Seventeen,” replied Polly.

Curly leered. “I suppose you’ve never been operated on?”

“Of course not.” Polly replied quite properly. “I’m absolutely convergent!”

“Come, come,” said Curly. “Let’s off to a decimal place I know and I’ll take you to the limit.”

“Never!” gasped Polly.

“Abscissa!” he swore, using the vilest oath he knew. His patience infinitesimal, he coshed her over the coefficient with a log until she was powerless. Curly removed her discontinuities. He stared at her significant places and began smoothing her points of inflection. She felt his hand tending toward her asymptotic limit. The convergence of her Fourier series would soon be gone forever. Poor Polly! The algorithmic method was now her only hope.

There was no mercy, for Curly was a Heaviside operator. Curly’s radius squared itself ( $r^2$ ). Polly’s loci quivered. He integrated by parts. He integrated by partial fractions. He integrated analytically. After he cofactor, he performed Runge-Kutta on her. The complex beast even went all the way around and did a contour integration. What an indignity to be multiply connected on her first integration! Curly continued operating until he satisfied her inductive hypothesis; then he exponentiated and became absolutely orthogonal. He attempted multiple integration, but to Polly’s good fortune, his elliptic section had become a degenerate conic.

When Polly got home that night, her mother observed she was no longer piece-wise continuous; she had been truncated in several places. Alas, it was too late to differentiate now. As the months went by, Polly’s denominator increased monotonically. Finally she went to L’Hospital and generated a small pathological hypergeometric function ( ${}_2F_1$ ) that left surds ( $\sqrt{\quad}$ ) all over the place and drove Polly to deviation.

The moral of our sad story is this: “If you want to keep your expressions convergent, never allow them a single degree of freedom.”

[This piece is reported to have appeared in *Michigan Technic* in March 1979, with no author credited. It had been in informal circulation for at least ten years before that. It has been embellished since then, including by today’s humble servant.]